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All References should be given in the text by quoting the surname of the author, followed by year of publication, *e.g.*, (Roy, 1958). The full Reference should be given in a list at the end, arranged alphabatically, as follows; Roy, S. B., 1958, *Ind. J. Phys.*, **32**, 323.

Line diagrams should be drawn on white Bristol board or tracing paper with black Indian ink, and letters and numbers inside the diagrams should be written neatly in capital type with Indian ink. The size of the diagrams submitted and the lettering inside should be large enough so that it is legible after reduction to one-third the original size. A simple style of lettering such as gothic, with its uniform line width and no serifs should be used, *e.g.*,

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Captions to all figures should be typed in a separate sheet and attached at the end of the paper.

The mathematical expressions should be written carefully by hand. Care should be taken to distinguish between capital and small letters and superscripts and subscripts. Repetition of a complex expression should be avoid by representing it by a symbol. Greek letters and unusual symbols should be identified in the margin. Fractional exponents should be used instead of root signs.

ON THE ELECTRONIC SPECTRA OF α-FLUORONAPH-THALENE IN THE LIQUID AND SOLID STATES*

S. B. BANERJEE

INDIAN ASSOCIATION FOR THE CULTIVATION OF SCIENCE, CALCUTTA-32

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ABSTRACT. The electronic absorption spectra of α-fluoronaphthalene in the liquid and solid states have been analysed and compared with the spectrum of the compound in the vapour state reported by previous workers. The substance in different states is found to exhibit two systems of bands in the 3200—3000 A.U. and 2900—2600 A.U. regions. The 0,0 band of the first system shows a shift of 442 cm^{-1} towards red when the vapour is lique-fied. When the liquid is solidified and cooled to −180°C, the 0,0 band is further displaced slightly towards longer wavelengths and the bands become sharper. The second system of bands due to the liquid consists of very broad bands and the 0,0 band is shifted towards red by about 1100 cm^{-1} from its positions in the spectrum due to the vapour. With solidi-fication of the liquid no further change is observed in this system.

INTRODUCTION

Naphthalene and its monoderivatives are known to exhibit two systems of absorption bands in the near ultraviolet region. It was observed in previous investigations (Deb, 1954; Banerjee, 1956) that with change of state and tempera-ture the bands of the two systems due to some monosubstituted naphthalene compounds undergo large changes indicating strong influence of intermolecular forces on the electronic energy state. Such an influence in the case of fluoronaph-thalenes had not been studied before. The present work was therefore undertaken to study the absorption spectra of α-fluoronaphthalene in the liquid and solid states and to compare the results with those due to the substance in the vapour state.

EXPERIMENTAL

The sample of α-fluoronaphthalene was obtained from Eastman Kodak Co. and was repeatedly distilled under reduced pressure before use. Thin films of the substance of thickness of the order of a few microns yielded the bands in the 2600—2900 A.U. region while much thicker film was required to obtain the bands of the first system on the longer wavelength side. The spectra were photographed on Ilford HP3 films with a Hilger E1 spectrograph. Microphotometric records

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of the spectrograms were taken with a Kipp and Zonen type Moll microphoto-meter. The wavelengths of the absorption peaks were determined with the help of microphotometric records of iron arc lines photographed on each spectrogram by the method described in an earlier paper (Banerjee, 1956).

RESULTS

The microphotometric records of the spectrograms are reproduced in Figs. 1 and 2 and the frequencies of the bands in cm^{-1} with probable assignments are

TABLE I

Absorption bands of α-fluoronaphthalene

Vapour* Ramamurty *et al.* (1957)		Liquid at 28°C Present author		Solid at −180°C Present author	
Wave number (cm^{-1}) and intensity	Assignment	Wave number (cm^{-1}) and intensity	Assignment	Wave number (cm^{-1}) and intensity	Assignment
31872 s	0,0	31430 s	0,0	31348 s	0,0
32281 m	0+409	32093 mb	0+663	31817 m	0+469
32545 m	0+673	32500 m	0+1070	32042 s	0+694
32710 vw	0+838	32866 s	0+1436	32416 m	0+1068
32918	0+1046	33530 s	0+663+1436	32786 s	0+1438
33307 m	0+1435			33092 w	0+1068+694
				33474 m	0+1438+694
	(Separation from 0,0)				
35222	0	34100 wb	0,0	34158 m	0,0
35399	177	35111 wvb	0+1011	34982 m	0+824
35683	461	36132 wvb	0+2×1011	34158 m	0+1375
36053	831			36370 w	0+1375+824
36341	1119				
36598	1376				
36807	1585				
37075	1853				
37512	2290				
38026	2804				
38481	3259				

*Bands corresponding to fundamental upper state frequencies only have been included.

First system (rows above the separation) and *Second system* (rows below) are indicated along the left margin.

given in Table I. The spectrum of the compound in the vapour state was studied earlier by Ramamurty *et al.* (1957), whose data have been included in the table for comparison. The compound in all the three states is found to exhibit two systems of bands in the regions 3200—3000Å and 2900—2600Å and these have been designated in this paper as the first and the second system respectively.
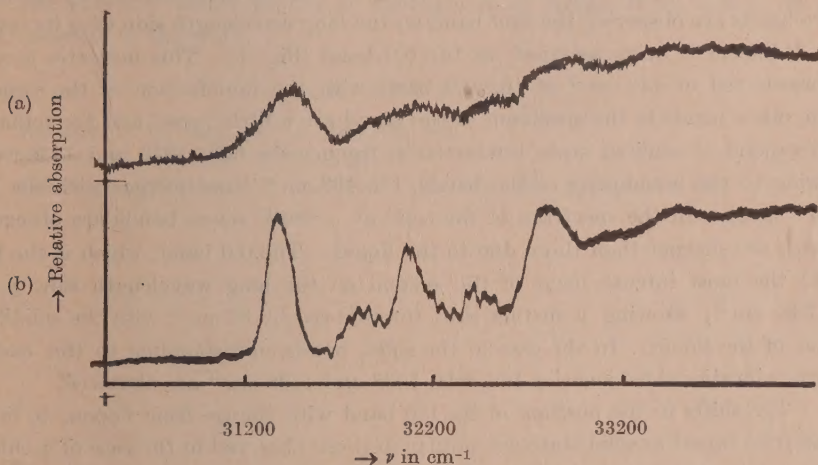
Fig. 1. Microphotometric records of the u. v. absorption spectra of α-fluoronaphthalene (first system).

(a) Liquid at 28°C   (b) Solid at — 180°C
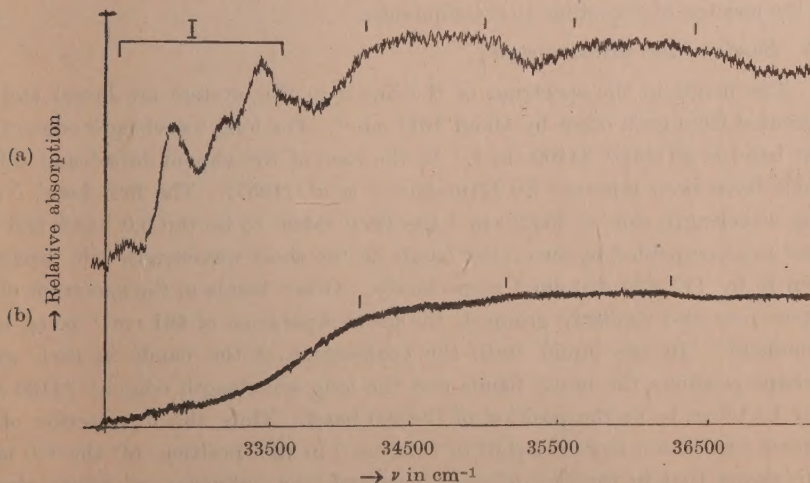
Fig. 2. Microphotometric records of the u. v. absorption spectra of α-fluoronaphthalene (second systeme).

(a) Solid at –180°C   (b) Liquid at 28°C

(The bands marked I belong to the 1st system)

DISCUSSION

(*i*) *Bands of the first system* :

The absorption spectrum of α-fluoronaphthalene in the vapour state shows a number of bands with the 0,0 band of this system at 31872 cm^{-1} (Ramamurty, Rao and Rao, 1957), the other band representing excited state vibrational frequencies 409, 673, 838, 1046 and 1435 cm^{-1}. In the spectrum due to the liquid five bands are observed, the first band on the long wavelength side with its centre at 31430 cm^{-1} being assigned as the 0,0 band (Fig. 1). This indicates a shift towards red of 442 cm^{-1} of the 0,0 band with the liquefaction of the vapour. The other bands in the spectrum of the liquid are a little broad and are found to correspond to excited state fundamental frequencies 663, 1070 and 1436 cm^{-1}. Owing to the broadening of the bands, the 409 cm^{-1} band merges with the 673 cm^{-1} band. In the spectrum of the solid at $-180°$C seven bands are observed, which are sharper than those due to the liquid. The 0,0 band, which is the first and the most intense band of the system on the long wavelength side, is at 31348 cm^{-1}, showing a further shift towards red by 82 cm^{-1} with the solidification of the liquid. In the case of the solid, bands corresponding to the excited state vibrational frequencies 469, 694, 1068 and 1438 cm^{-1} are observed.

The shifts in the position of the 0,0 band with change from vapour to liquid and from liquid to solid state are similar to those observed in the case of α-chloro- and α-bromonaphthalene (Deb, 1954). This indicates that in the case of this molecule also the excited electronic energy level is lowered by the intermolecular field in the condensed phases of the substance in the same way as observed in the spectra of the other two compounds.

(*ii*) *Bands of the second system* :

The bands in the spectrum of the liquid in this system are broad and are separated from each other by about 1011 cm^{-1}. The long wavelength edge of the first band is at about 34100 cm^{-1}. In the case of the vapour broad and diffuse bands have been reported by Ramamurty *et al.* (1957). The first band on the long wavelength side at 35222 cm^{-1} has been taken to be the 0,0 band and this band is accompanied by two other bands on the short wavelength side separated from it by 177 and 461 cm^{-1} respectively. Other bands in the spectrum of the vapour are also similarly grouped, the mean separation of 461 cm^{-1} being quite prominent. In the liquid state the coalescence of the bands in each group perhaps produces the broad bands and the long wavelength edge at 34100 cm^{-1} may be taken to be the position of the 0,0 band. Thus, the liquefaction of the vapour results in a large red shift of 1122 cm^{-1} in the position of the 0,0 band. This shows that in the case of this compound the influence of intermolecular field in the liquid state is greater on the second system than on the first. In the spectrum of the solid at $-180°$C, the bands are still broad and the position of the 0,0 band remains almost unaltered with solidification.

Thus the main changes in the spectra of both the systems are found to take place with change from the vapour to the liquid state. This probably indicates formation of strongly associated groups of the molecules in the liquid state due to the presence of highly active fluorine atom in the molecule.

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ON THE F₂-REGION OF THE IONOSPHERE

S. DATTA

INSTITUTE OF RADIO PHYSICS AND ELECTRONICS, CALCUTTA

ABSTRACT. Calculations of the total electron production rate in a column of unit cross-section extending from the "bottom" to the maximum electron density height and of the mean production rates in different parts of the F_2-region are made with the help of the attachment coefficient model suggested by Ratcliffe *et al.* (1956). It is found that the results are consistent with those expected from the hypothesis of Bradbury for the formation of the F2-region. The calculations are made by the method suggested by the author (1958).

INTRODUCTION

The diurnal variation of electron density N at a given height in an ionized region, neglecting the effects of movements, is governed by the equation

$$\frac{dN}{dt} = q - KN^\eta \qquad \cdots \quad (1)$$

where q = electron production rate

           K = electron loss coefficient

and $\eta = 1$ or 2 depending on the electron loss process.

In the E and F_1 regions the effects of movements are negligible. For these regions Eq. (1) may be utilised for calculating the production rate from the experimental values of $\frac{dN}{dt}$ and N provided the loss process and the loss coefficient K are known. In the F_2 region, however, movements of the electrons produce changes in electron density at a rate comparable to dN/dt of Eq. (1). Computations of production rate with the help of this equation cannot, therefore, yield reliable values. It is also not possible to take account of the effects of the movements in the computation as little is known about their nature and magnitude. Further, the height of the F_2 region has large diurnal variations and its thickness also changes considerably at times. These difficulties notwithstanding, study of the production rates in different levels of the thick F_2 region, is desirable for testing the theories of the formation of the F_2 region.

A method of computation of such production rates was therefore proposed by the author of this paper. The method consists in dividing a column of unit cross-section of the region extending from the "bottom" to the height of maximum electron density into four sections of equal length. Mean production rate in

each of these sections (same as mean production rate in four different parts of the thick F_2 region) is computed from the diurnal variation of the total number of electrons in it. The total production rate in the "unit column" of the F_2 region and the mean production rate in the region are also computed. Such computations obviously minimise the effects of layer movement as a whole, movements of electrons, and layer contractions and dilutions due to thermal changes. There is, of course, the proviso that the fraction of the total number of electrons present in each of the four "unit" sections is not changed. Mean hourly values of N at a series of heights in the F_2 region over Slough for the months of January and March, 1950 on international quiet days were utilised for the computation. "Tables of F_2-layer Electron Density on International Quiet Days" computed by Schmerling and Thomas (1955) were obtained from the Radio Group, Cavendish Laboratory, Cambridge, England. The height variation of the attachment coefficient as suggested by Ratcliffe *et al*. (1956) from night-time observations over Slough, Watheroo and Huancayo, was utilised.

In the paper of the author referred to above (Datta 1958) the diurnal variations of the production rates (as mentioned above) were delineated for the month of January, 1950. In the present paper, computations are made of the same parameters for the month of March of the same year. Values of the mean production rates have also been calculated on the basis of Bradbury's (1938) hypothesis for the formation of the F_2 region. These are compared with the results for the two months, January and March, 1950 obtained by the suggested method of analysis.

RESULTS

Fig. 2(a) depicts the diurnal variation of the total number of electrons n in a column of unit cross-section extending from the "bottom" to the height of maximum electron density in the F_2 region over Slough for the month of March, 1950. Fig. 2(b) shows the diurnal variations for the same month of n_r ($r = 1, 2, 3, 4$), the total number of the electrons in each of the four sections of the column taken in order from the "bottom". Hourly values of the number of electrons and the loss rates in each of the sections were calculated by Simpson's rule for numerical integration. Loss rates were calculated with the help of the height variation of the attachment coefficient as suggested by Ratcliffe *et al*. (1956) and extrapolated to a height of 200 Km. (Fig. 1). To obtain half hourly values of $\left(\dfrac{dn_r}{dt} \right)$ and n_r a linear change between the hourly values of n_r was assumed. A quassi-equilibrium condition in each of the columns was assumed from 10 hrs. to 14 hrs. Such an assumption is justified in the neighbour-

hood of the midday when loss rate and production rate are much larger than

$\left(\dfrac{dn_r}{dt} \right)$. The diurnal variations of mean production rate for the month of March,
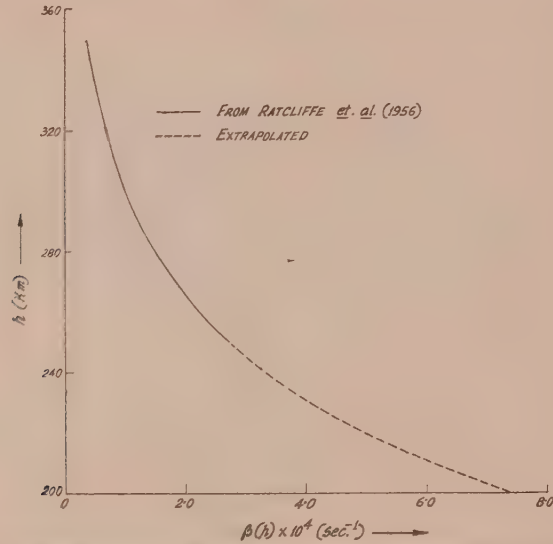
Fig. 1.   Height variation of the attachment coefficient suggested by Ratcliffe *et al.* (1956).
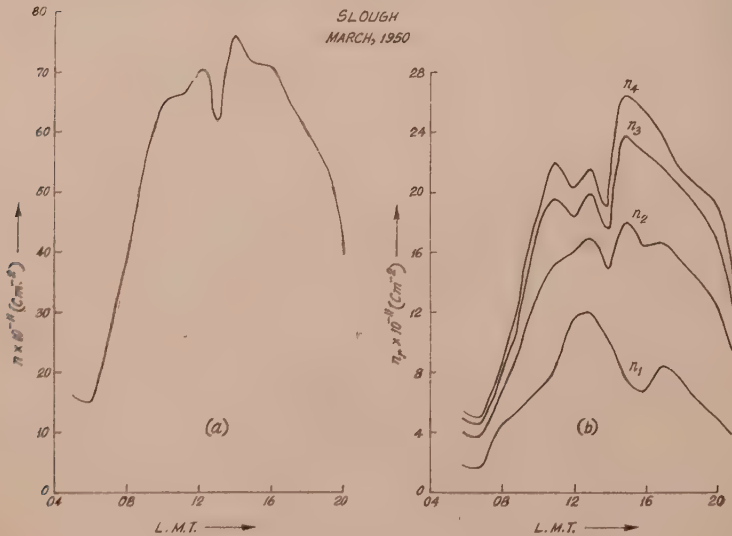
Fig. 2.   (a) Diurnal variation of n in the F_2 region for the month of March, 1950.

           (b) Diurnal variations of n_r ($r = 1,2,3,4$) in the F_2 region for the month of March, 1950.

1950 over Slough are shown in Fig. 3. The diurnal variation of the mean production rate (q) and the total production rate (Q) in the whole column of unit
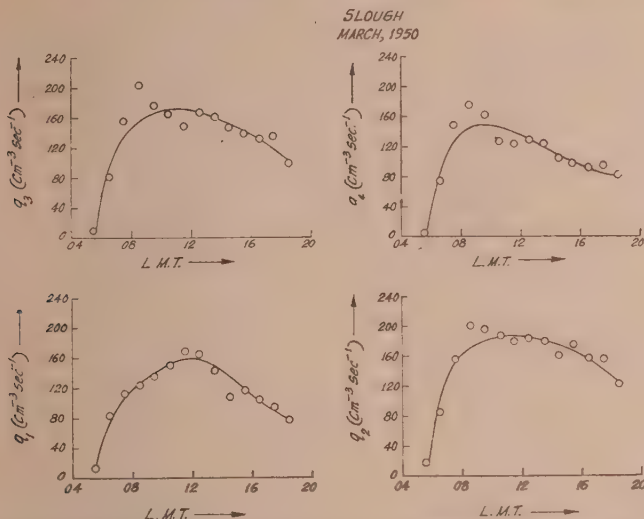
Fig. 3.   Diurnal variations of q_r ($r = 1,2,3,4,$) in the F_2 region for the month of March, 1950.

cross-section extending from the "bottom" to the maximum electron density height in the F_2 region for the months of January and March, 1950 over Slough, are shown by broken lines in Fig. 5 and Fig .6 respectively.
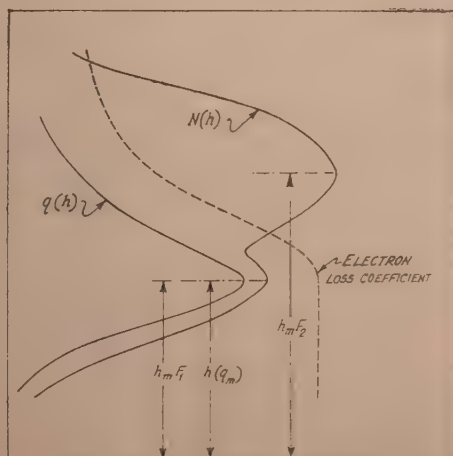
Fig. 4.   Diagram illustrating the hypothesis of Bradbury for the formation of the F_2 region.

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SLOUGH

JAN. 1950
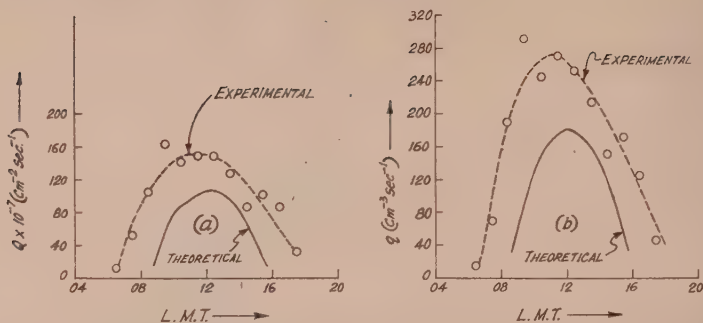
Fig. 5. Diurnal variation of Q and q (Experimental and theoretical) in the F_2 region for the month of January, 1950.

SLOUGH

MARCH, 1950
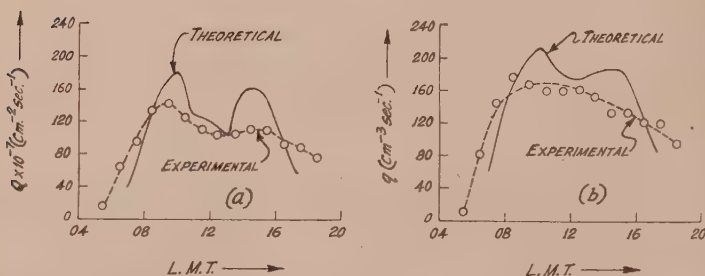
Fig. 6. Diurnal variation of Q and q (experimental and theoretical) in the F_2 region for the month of March, 1950.

TABLE I

Half hourly values of q_r ($r = 1, 2, 3, 4$), Q and q.

January 1950—Slough

Hour (L.M.T.)	q_1 cm^{-3}.sec.$^{-1}$	q_2 cm^{-3}.sec.$^{-1}$	q_3 cm^{-3}.sec.$^{-1}$	q_4 cm^{-3}.sec.$^{-1}$	$Q \times 10^{-7}$ cm^{-2}.sec.$^{-1}$	q cm^{-3}.sec.$^{-1}$
0630	14	17	16	14	13	15
0730	37	70	84	89	51	70
0830	128	227	200	203	106	190
0930	205	332	314	317	163	292
1030	180	303	273	222	141	245
1130	193	319	306	262	151	270
1230	183	311	283	230	151	252
1330	173	255	231	191	127	213
1430	115	195	166	119	86	149
1530	135	208	190	151	102	171
1630	94	154	136	109	86	123
1730	60	69	40	7	33	44

TABLE II

Half hourly values of q_r ($r = 1, 2, 3, 4$), Q and q.

March 1950—Slough

Hour (L.M.T.)	q_1 cm^{-3}.sec.$^{-1}$	q_2 cm^{-3}.sec.$^{-1}$	q_3 cm^{-3}.sec.$^{-1}$	q_4 cm^{-3}sec.$^{-1}$	$Q \times 10^{-7}$ cm^{-2}.sec.$^{-1}$	q cm^{-3}.sec.$^{-1}$
0530	12	18	10	4	11	11
0630	83	87	82	74	64	82
0730	113	160	159	150	96	145
0830	123	200	206	174	134	176
0930	135	196	177	161	142	167
1030	151	188	166	128	120	158
1130	169	179	150	124	106	156
1230	164	184	160	127	102	159
1330	144	181	160	122	108	152
1430	108	160	148	105	111	130
1530	114	175	139	96	109	131
1630	104	158	133	92	93	122
1730	94	160	138	97	90	122
1830	76	122	99	81	76	95

COMPARISON WITH THEORY

For comparison, the values of Q, q and q_r (mean production rate in the rth column) may be calculated from the hypothesis of Bradbury (1938) as follows :

If a gas of constant scale height H is ionized by monochromatic radiation, then the value of $q(h)$, when the solar zenithal angle is χ, is given by the expression due to Chapman (1931).

$$q(h) = q_0 \exp (1 - Z - e^{-Z} \sec \chi) \qquad \ldots \quad (2)$$

where $Z = \dfrac{h - h_0}{H}$ and h_0 is the height of maximum production rate q_0 when $\chi = 0$.

According to Bradbury, F_1 and F_2 layers are both produced by the same ionization process. The ionizing radiation for the process, acting on a single ionizable gas constituent, has the height $h(q_m)$ of peak production rate near the level of F_1 layer peak ($h_m F_1$); and, the F_2 layer peak ($h_m F_2$) is formed as a result of the rapid decrease of the electron loss coefficient above F_1 layer, the loss coefficient in the F_1 layer being independent of height. The hypothesis of Bradbury is illustrated in Fig 4. According to this hypothesis, when the solar zenith angle is χ, the value of Q, the total electron production in a column of unit cross-section of the F_2 region extending from the "bottom" ($h_o F_2$) to the height of maximum density ($h_m F_2$) is given by

$$Q = \int_{h_o F_2}^{h_m F_2} q(h) \, . \, dh = q_0 H \cos \chi [\exp (1 - e^{-Z_2} \sec \chi) - \exp (1 - e^{-Z_1} \sec \chi)] \quad \ldots (3)$$
$$= q \, . \, T$$

where
$$Z_2 = \frac{h_m F_2 - h_0}{H}$$

$$Z_1 = \frac{h_0 F_2 - h_0}{H}$$

$$T = h_m F_2 - h_0 F_2$$

q = mean production rate in the F_2 layer and q_0 = peak production rate.

The value of the peak production rate (q_0) in the F_1 region when the sun's rays are vertical may be estimated for the quasi-equilibrium conditions (production rate = loss rate) from the expression,

$$q_0 = \frac{\alpha N_m{}^2 F_1}{\cos \chi} \qquad \qquad \dots \ (4)$$

where $N_m F_1$ = maximum electron density in the F_1 layer when the solar zenithal angle is χ and α = coefficient of recombination in the F_1 layer.

Assuming $\alpha = 5 \times 10^{-9}$ cm³. sec. ⁻¹ [Bates and Massey (1946), Rydbeck (1946)], Ratcliffe *et al.* found from an analysis of Slough and world-wide data that

$$q_0 = 280 \ (1 + 1.4 \times 10^{-2} \bar{R}) \ \text{cm}^{-3}. \ \text{sec.} \ ^{-1} \qquad \dots \ (5)$$

where $\bar{R}$ is the monthly average relative Zürich sunspot number.

It has been suggested (Bates and Massey 1948 and other authors) that the ionizable gas in the F region is atomic oxygen. The value of the scale height (H) of atomic oxygen is about 50 km. between 200 km. and 400 km. according to the R model (based on rocket results) given by Bates (1954). This value of the scale height is consistent with the value ($H = 45$ km.) accepted by Ratcliffe *et al.* after a critical examination of the experimental results and Bradbury's hypothesis. The value of h_0 accepted by Ratcliffe *et al.* is 180 km. With these values of H and h_0 (45 km. and 180 km. respectively) the hourly values of the total production rate (Q) in the unit column and mean production rate (q) in the F_2 layer have been calculated with the help of Eq. (3).

For comparison, the theoretical hourly values of Q and q (calculated on the assumption of Bradbury's hypothesis) and the hourly experimental values (from the mean curves in Fig. 5 and Fig. 6) are shown in Table III and Table IV.

To show how the values of the mean production rates q_r should fall with increasing order of the sectional column (first column being the lowest one) according to the hypothesis of Bradbury, values of the same have been calculated theoretically. This can be done with the help of the equation

$$q_r = \frac{x}{T} q_0 H \cos \chi [\exp (1 - e^{-Z_{2r} \sec \chi}) - \exp (1 - e^{-Z_{1r} \sec \chi})] \qquad \dots \ (6)$$

where Z_{2_r} = reduced height of the upper boundary of the rth column,

Z_{1_r} = reduced height of the lower boundary of the r th column.

TABLE III

Hourly experimental and theoretical values of Q and q.

January 1950—Slough

Hour (L.M.T.)	$Q \times 10^{-7}$ cm^{-2}.sec.$^{-1}$ (Experimental)	$Q \times 10^{-7}$ cm^{-2}.sec.$^{-1}$ (Theoretical)	q cm^{-3}.sec.$^{-1}$ (Experimental)	q cm^{-3}.sec.$^{-1}$ (Theoretical)
09	120	27	220	51
10	144	81	256	133
11	152	94	272	168
12	150	106	260	183
13	140	102	238	170
14	126	85	206	144
15	100	46	168	84

TABLE IV

Hourly experimental and theoretical value of Q and q

March 1950—Slough

Hour (L.M.T.)	$Q \times 10^{-7}$ cm^{-2}.sec.$^{-1}$ (Experimental)	$Q \times 10^{-7}$ cm^{-2}.sec.$^{-1}$ (Theoretical)	q cm^{-3}.sec.$^{-1}$ (Experimental)	q cm^{-3}.sec.$^{-1}$ (Theoretical)
07	80	40	116	61
08	118	101	150	149
09	140	156	162	184
10	132	180	168	212
11	116	128	168	191
12	106	120	164	174
13	104	103	158	178
14	108	157	150	187
15	110	158	138	186
16	104	117	128	146
17	92	67	116	94

Values of the mean production rates of the four sectional columns at midday [1200 hr. (L.M.T.)] so calculated are shown below in Table V for the month of January 1950 and March 1950 at Slough. The calculations were done with values of H, h_0 and q_0 as stated earlier. It is to be noted that $q_1 > q_2 > q_3 > q_4$. This is as it should be, because, as the peak production rate is assumed to be near the F_1 peak height, the mean production rate should decrease with increasing height of the column in the F_2 region.

TABLE V

Theoretical values of q_r ($r = 1, 2, 3, 4$) according to the hypothesis of Bradbury

Hr.—1200 (L.M.T.)—Slough

1950 Month	q_1 cm^{-3}.sec.$^{-1}$	q_2 cm^{-3}.sec.$^{-1}$	q_3 cm^{-3}.sec.$^{-1}$	q_4 cm^{-3}.sec.$^{-1}$
January	201	197	173	166
March	247	206	129	124

DISCUSSION

The computations of the diurnal variations of the mean production rates in the different sections of the F_2 region by the method of analysis, suggested by the author, taking into account the height variation of the attachment coefficient after Ratcliffe *et al.* give consistent results. The diurnal variations are found to be more regular and symmetrical about noon for the winter month (January) than for the equinoxial month (March). The calculated values of the mean production rates for both the months as given in Table I and Table II, show that $q_2 > q_3 > q_4$ for most of the half hourly values. This is in accordance with Table V, except that the experimental q_1's are generally less instead of being greater than q_2's. This may be due to the uncertainty in the correct determination of electron density near the "bottom" of the F_2 region. This uncertainty affects the computed values of n_1. Another possible reason for 'this discrepancy may be due to the fact that the loss rates in the first column were often calculated from the extrapolated values (below 250 km) of attachment coefficient.

It may be noted from Table III that the theoretical values of Q and q are lower than the experimental values. The difference is smaller for the month of March than for the month of January. It must be mentioned in this connection that the magnitudes of Q and q, calculated theoretically on the assumption of Bradbury's hypothesis, depend on the assumed values of q_0, h_0 and H. The assumed value of q_0, again in its turn, is proportional [Eq. (4)] to the assumed values of the recombination coefficient (α) in the F_1 region. The value of q_0 utilized for the theoretical computation was obtained by Ratcliffe *et al.* by assuming $\alpha = 5 \times 10^{-9}$ cm^3 sec.$^{-1}$. Higher values $\alpha = 8 \times 10^{-9}$ cm^3. sec.$^{-1}$, however, has been suggested (Minnis, 1955). If this latter value is accepted, the theoretically calculated values of q and Q will be increased by about 50% provided the values of H and h_0 are unaltered.

It may thus be concluded that with the height variation of the attachment coefficient suggested by Ratcliffe *et al.* the results as obtained by the method of analysis adopted are consistent with the hypothesis of Bradbury for the formation of the F_2 region.

ACKNOWLEDGMENTS

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INTERMOLECULAR POTENTIAL OF HELIUM

A. K. BARUA

INDIAN ASSOCIATION FOR THE CULTIVATION OF SCIENCE, CALCUTTA-32, INDIA

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ABSTRACT. The potential energy function for He–He interaction has been obtained on the exp 6–8 model which contains the dipole-quadropole interaction term in addition to the dipole-dipole term in the attractive potential, by fitting in second virial and Joule-Thomson coefficient data over extensive range of temperatures. The necessary quantum corrections have been considered. Excellent agreement is obtained between the values calculated from the potential energy function determined and the experimental data. On the whole the exp. 6–8 potential gives better fit with the second virial and the Joule-Thomson coefficient data than either the exp-6 or the Lennard-Jones (12 : 6) potential.

1. INTRODUCTION

The potential energy function of molecules may be determined in two different ways. First, the potential energy function may be calculated directly from the atomic structure provided the molecules are sufficiently simple systems. The second method is to assume a carefully chosen potential form involving a number of constants whose values may be determined from accurate experimental data. Helium atom is particularly interesting in this respect for its simple structure and comparatively large quantum effects which play an important part up to temperatures of the order of 400°K.

The direct calculation of the interaction potential between two helium atoms results in a potential energy function of the form

$$\phi(r) = a \exp(-br) - (cr^{-6} + dr^{-8} + er^{-10} + \dots) \qquad \dots \quad (1)$$

where $\phi(r)$ is the potential energy between two molecules separated by a distance r. Slater and Kirkwood (1931) and Kirkwood and Keyes (1931) considered only the r^{-6} term in the attractive potential which represents dipole-dipole interaction. But Margenau (1931; 1939) showed that though the dipole interactions are the only appreciable ones when the molecules are far apart, the higher poles are to be considered at distances of the order of the kinetic theory radius. His calculations proved that although contributions from the r^{-10} term representing quadrupole—quadrupole interaction may be neglected, the r^{-8} term representing dipole-quadrupole interaction contributes a sizeable fraction of the total interaction energy. Margenau obtained for He the potential form

$$\phi(r) = \left(770e^{-r/0\cdot217} - 560e^{-r/0\cdot187} - \frac{1.39}{r^6} - \frac{3.0}{r^8} \right) \times 10^{-12} \text{ erg.} \qquad \dots \quad (2)$$

More recently the repulsive part of the interatomic potential of He has been calculated by Rosen (1950), Griffing and Wehner (1955) and others more accurately. It has been observed by Yntema and Schneider (1950) that although the potential form derived by Kirkwood and Keyes (1931) gives somewhat fair agreement with their second virial data, generally the intermolecular potentials calculated directly from the atomic structures fail to explain the experimental data satis- factorily.

The alternative method of fitting the experimental data to suitably chosen potential forms has been tried by many workers (Buckingham, 1938; Massey, 1939; 1941; Lunbeck 1951, etc.). Secondvirial coefficient data have been fitted to the Buckingham and Lennard-Jones (12 : 6) potentials. Zero-pressure Joule-Thomson coefficient data have been used by Hirschfelder (1938) *et al*, to obtain the potential energy function for He on the Lennard-Jones (12 : 6) model. Recently Mason and Rice (1954) have fitted second virial and viscosity data to the modified exp-6 potential. Their determination of the intermolecular potential has the drawbacks that the r^{-8} term is not included in the potential form and both the first and the second quantum corrections have been taken from the exp 6-8 and the Lennard-Jones (12 : 6) potentials respectively which is not strictly justified especially as the first quantum correction for He is quite large at the lower temperatures. Moreover, it appears that at comparatively low temperatures the agreement between the experimental and the calculated values of the second virial coefficient is not very good and this disagreement is expected to be larger at the lower temperatures. Amdur (1954) and his co-workers have used molecular beam scattering method to obtain experimentally the repulsive part of the potential for He-He interactions. The theoretically calculated values of the repulsive energy for He-He interaction are consistently higher than the experimental values of Amdur *et al*.

Yntema and Schneider (1950) have tried to fit their experimental second virial data to an empirical potential of the form :

$$\phi(r) = b \exp(-r/\rho) - (c_1/r^6) - (c_2/r^8). \qquad \ldots \quad (3)$$

Though good agreement has been obtained with their experimental data they did not consider the quantum corrections.

In view of the above considerations, the most appropriate potential form for He seems to be exp 6-8 potential proposed by Buckingham and Corner (1947) which is given by

$$\phi(r) = \epsilon \left[g_1(\alpha, \beta) \exp \alpha \left(1 - \frac{r}{r_m}\right) - g_2(\alpha, \beta) \left(\frac{r_m}{r}\right)^6 \left(1 + \beta \left(\frac{r_m}{r}\right)^2\right) \right] \text{ for } r \geqslant r_m,$$

$$\ldots \quad (4)$$

3

$$\phi(r) = \epsilon \left[\ g_1(\alpha, \beta) \exp \alpha \left(1 - \frac{r}{r_m} \right) - g_2(\alpha, \beta) \left(\frac{r_m}{r} \right)^6 \left(1 + \beta \left(\frac{r_m}{r} \right)^2 \right) \right.$$

$$\left. \exp 4 \left(1 - \frac{r_m}{r} \right)^3 \right] \ \text{for} \ r \leqslant r_m \ \dots \quad (5)$$

$$g_1(\alpha, \beta) = (6 + 8\beta)/[\alpha(1 + \beta) - (6 + 8\beta)]$$

$$g_2(\alpha, \beta) = \alpha/[\alpha(1 + \beta) - (6 + 8\beta)]$$

where ϵ is the depth of the potential well, r_m the value of r for which $\phi(r)$ has its minimum value, α is the parameter which measures the steepness of exponential repulsion.

This potential form has the advantage that it not only includes an exponential repulsion term and the r^{-6} and r^{-8} terms but also the attractive term is provided with an exponential that prevents the appearance of a spurious maximum having no physical significance in the case of the exp-6 potential at $r \approx 0.25 \ r_m$. The first quantum correction for the second virial and the Joule-Thomson coefficient have been calculated accurately for the exp 6-8 model. In this paper second virial data over an extensive range of temperature and the zero-pressure Joule-Thomson coefficient data have been fitted to the exp 6-8 potential. It is expected that the potential form thus derived will be more reliable than those hitherto obtained. Force parameters on the exp 6-8 model has been determined by Corner (1948) for A, Ne and for Kr, Xe and C_2H_4 by Barua (1959). In order to reproduce their low temperature diffusion data for He, Buckingham and Scriven (1952) have used Eq. (4) only and have chosen empirically the parameters $\alpha = 13.5$, $\beta = 0.2$, $\epsilon/K = 10.18°K$, $r_m = 2.943Å$.

2. DETERMINATION OF THE PARAMETERS

Kirkwood (1933) and Uhlenbeck and Beth (1936) have shown that the second virial coefficient $B(T)$ together with the quantum corrections may be written as

$$B(T) = \left[\ B_{Cl}(T) + \frac{h^2}{m} \ B_I(T) + \left(\frac{h^2}{m} \right)^2 B_{II}(T) + \dots \ \right] \quad \dots \quad (6)$$

where $B_{Cl}(T)$ is the classical expression for the second-virial coefficient and $B_I(T)$ and $B_{II}(T)$ are respectively the first and second quantum corrections, m is the mass of the molecule and h is the Planck's constant.

On the exp 6-8 model

$$B_{Cl}(T) = 2\pi N r_m{}^3 F_0(\alpha; \beta; T^*), \quad \dots \quad (7)$$

$$B_I(T) = (N r_m/kT) F_1(\alpha; \beta; T^*), \quad \dots \quad (8)$$

$$T^* = kT/\epsilon \quad \dots \quad (9)$$

where N is the Avogadro's number and k is the Boltzmann's constant. The second quantum correction $B_{11}(T)$ has not been evaluated on the exp 6-8 model. Since the contribution of $B_{II}(T)$ is not large (its contribution to the total $B(T)$ at 100°K is about 1%), as has been done by Mason and Rice (1954), we may take the value of $B_{,}(T)$ calculated by de Boer and Michels (1938) on the Lennard-Jones (12 : 6) model. Due to this uncertainty in the second quantum correction Eq. (6) cannot be used below 60°K.

Similarly, the zero pressure Joule-Thomson coefficient may be written as

$$\mu^0 = \left[\ \mu^0_{cl} + \frac{h^2}{m} \mu_I{}^0 + \left(\frac{h^2}{m} \right)^2 \mu_{11}{}^0 + \ldots \right], \qquad \ldots \ (10)$$

and on the exp 6-8 model

$$\mu_{cl}^0 = (2\pi N r_m{}^3/C_p{}^0)G_0(\alpha; \ \beta; \ T^*) \qquad \ldots \ (11)$$

$$C_p{}^0 \mu_I{}^{\ 0} = (N r_m/kT)G_1(\alpha; \beta; T^*) \qquad \ldots \ (12)$$

where $C_p{}^0$ is the value of the zero-pressure molar specific heat. The functions F_0, G_0, F_1, G_1 have been tabulated by Buckingham and Corner (1947) for $\alpha = 12.5$ to $\alpha = 14.5$ and for $\beta = 0$ and $\beta = 0.2$.

The force parameters α, β, ϵ/k, r_m can be determined from the experimental $B(T)$ data by the method of translation of axes (Mason and Rice, 1954; Srivastava, 1957). In order to consider the quantum corrections, first one of the two values of β (either $\beta = 0$ or $\beta = 0.2$) is assumed and the quantum corrections B_I and B_{II} are taken to be zero and a set of values α, β, ϵ/k, r_m is obtained. With these parameters B_I and B_{II} are calculated which are then substituted in Eq. (6) to give α, β, ϵ/k, r_m to the second approximation. This procedure is to be repeated till the force parameters do not vary appreciably.

In Table I the force parameters obtained on the exp 6-8 model are given. β_M refers to the value of β as obtained by Margenau (1939) and α, ϵ/k, and r_m are assumed to vary linearly with β to find values corresponding to β_M. For the sake of comparison the force parameters for the exp-6 and the Lennard Jones (12 : 6) potentials are also given in Table I.'

3. COMPARISON WITH EXPERIMENT

An obvious test of the reliability of the intermolecular potential determined is a comparison of the values calculated from the potential energy function with the experimental data. A further test is the ability to reproduce more than one property with the same set of force parameters. The experimental second virial data over a temperature range of 90.2°K to 1473°K have been compared with the calculated values on the exp 6-8 model and the results are given in table II.

TABLE I

Potential parameters for He

Potential form	α	β	$\varepsilon/k°K$	$r_m Å$	Ref. for data	
					2nd Virial	J–T Coeff.
Exp. 6–8	11.4	0	9.45	3.13		
(Buckingham	11.2	0.2	9.89	3.13	v	j
Corner)	11.2	$0.16(=\beta_M)$	9.82	3.13		
Exp. 6*	12.4	....	9.16	3.136		
L–J (12 : 6)*	....	....	10.22	2.869		

*   (From Mason and Rice, 1954)

v Holborn and Otto (1926)
   Michels and Wouters (1941)
   Schneider and Duffie (1949)
   Yntema and Schneider, (1950)

j. Roebuck and Osterberg (1933; 1934).

The second quantum correction which becomes important at the lower temperatures have been taken from the calculations of de Boer and Michels (1938) on the Lennard-Jones (12 : 6) model.   Excellent agreement is obtained with the experimental data over the whole range of temperatures.   Exp 6-8 potential gives better fit than the exp-6 potential (particularly at the lower temperatures) excepting at the highest temperatures.   Throughout the temperature range exp 6-8 potential is found to reproduce the experimental data better than the Lennard-Jones (12 : 6) potential and this is very much marked at the higher temperatures. This confirms Mason and Rice's (1954) observation that helium atom is actually softer than that given by the Lennard-Jones (12 : 6) model.

The experimental and calculated values of the zero-pressure Joule-Thomson coefficient on the exp 6-8 and the Lennard-Jones (12 : 6) models are given in table III.   Temperatures below $172°K$ have not been considered, for, at lower temperatures second quantum correction becomes important.   The agreement on the exp-6-8 model is remarkably better than that obtained on the L-J (12 : 6) model. This single case perhaps cannot be taken as a proof of the superiority of the potential form with exponential repulsion term over the Lennard-Jones (12 : 6) potential. The fact that almost the same degree of agreement is obtained with the experimental data for both $\beta = 0$ and $\beta = 0.2$ confirms the observation of the earlier workers (Corner, 1948; Barua, 1959) that these equilibrium properties are not sensitive enough to measure the relative importance of the r^{-6} and r^{-8} terms. The third virial coefficient which is quite sensitive to the potential form chosen may serve the purpose.   But no calculation for the third virial coefficient has been done for either exp-6 or exp-6-8 potentials.

TABLE II

Comparison of experimental and calculated values of $B(T)$ of He.

T°K	$B(T)$ in cc/mole	exp 6–8		exp–6	Lennard-Jones (12 : 6)
		$\beta=0$	$\beta=0.2$		
90.2	10.45[a]	10.55	10.34	11.40	10.43
123.2	11.42[a]	11.36	11.27	11.66	10.76
173.2	11.92[a]	11.82	11.78	12.34	11.31
223.2	11.94[a]	11.90	11.84	12.21	11.59
273.2	11.86[a] 11.87[b] 11.77[c]	11.70	11.69	12.11	11.50
298.2	11.74[b]	11.60	11.57	11.99	11.46
323.2	11.74[b] 11.58[b]	11.47	11.53	11.86	11.39
348.2	11.43[b]	11.37	11.42	11.73	11.30
373.2	11.39[a] 11.35[b] 11.42[c]	11.25	11.30	11.60	11.22
398.2	11.24[b]	11.14	11.16	11.48	11.07
423.2	11.07[b]	11.07	11.09	11.36	11.01
473.2	11.07[a] 11.08[c]	10.95	10.98	11.12	10.90
573.2	10.50[a] 10.76[c]	10.30	10.44	10.71	10.65
673.2	10.14[a] 10.45[c]	10.01	10.05	10.33	10.41
773.2	10.14[c]	9.74	9.75	9.99	10..22
873.2	9.82[c] 9.80[d]	9.52	9.57	9.68	10.03
1073.2	9.17[d]	9.00	9.11	9.11	9.63
1273.2	8.66[d]	8.78	8.85	8.69	9.50
1473.2	8.19[d]	8.49	8.66	8.33	9.27

(a) Holborn and Otto (1926) ; (b) Michel and Wouters (1941) ; (c) Schneider and Duffie (1949) ; (d) Yntema and Schneider (1950).

TABLE III

Comparison of experimental and calculated values of μ^0 of He.

T°K	μ^0 in deg. at m^{-1}	exp 6-8		Lennard-Jones (12 : 6)
		$\beta = 0$	$\beta = -0.2$	
172.1	−0.0557	−0.0556	−0.0556	−0.0506
273.1	−0.0599	−0.0608	−0.0611	−0.0579
297.5	−0.0601	−0.0613	−0.0614	−0.0587
374.2	−0.0621	−0.0617	−0.0622	−0.0593
472.5	−0.0616	−0.0615	−0.0624	−0.0593
575.5	−0.0568	−0.0596	−0.0608	−0.0589

4. COMPARISON WITH THEORETICALLY CALCULATED POTENTIAL ENERGY FUNCTIONS

It has been shown by Margenau (1939) that the interaction energy of two He atoms may be represented as the sum of four terms.

$$\phi(r) = \phi^{(val)} + \phi^{(ex, 2)} + \phi^{(dis, 6)} + \phi^{(dis, 8)} \qquad \ldots \text{(13)}$$

where $\phi^{(val)}$ is the valence energy of repulsion obtained by the first-order perturbation calculation. $\phi^{(ex, 2)}$ represents the second-order exchange terms and becomes important at distances where long and short-range forces meet. It is obtained in the second-order perturbation calculation of dispersion energy. $\phi^{(dis, 6)}$, $\phi^{(dis, 8)}$ are respectively the dispersion energy terms varying as the inverse sixth power and the inverse eighth power of separation.

The $\phi^{(val)}$ term has been evaluated by various workers. Slater (1928) and Rosen (1950) and Sakamoto and Ishiguro (1956) used the valence bond method and later workers (Griffing and Wehner, 1955; Huzinaga, 1957) have applied LCAO MO method. For comparison with the potential energy curve obtained on the exp-6-8 model we shall take $\phi^{(val)}$ calculated by Sakamoto and Ishiguro (1956) which is in good agreement with that of Griffing and Wehner (1955). The $\phi^{(ex, 2)}$ and $\phi^{(dis, 6)}$ terms have been obtained accurately by Margenau by the second-order perturbation calculation of dispersion energy. The $\phi^{(dis, 8)}$ term has been obtained by Page (1938) by the variational method. Hence the potential energy function for He-He interaction becomes

$$\phi_{SI-M-P}^{(r)} = \phi_{SI}^{(val)} + \phi_{M}^{(ex, 2)} + \phi_{M}^{(dis, 6)} + \phi_{P}^{(dis, 8)} \qquad \ldots \text{(14)}$$

$$= \left[655e^{-4.469r} \quad 560e^{-5.33r} - \frac{1.39}{r^6} - \frac{3.0}{r^8} \right] \times 10^{-12} \text{ erg.} \qquad \ldots \text{(15)}$$

The potential energy curve obtained from Eqs. (4) and (5) on the exp-6-8 model (taking the value of $\beta = \beta_M$) by utilising the force parameters determined in the present investigation together with the theoretically obtained curve from Eq. (15) is shown in Fig. 1. For comparison the potential energy curve on the exp-6 model obtained by Mason and Rice (1954) is also shown.
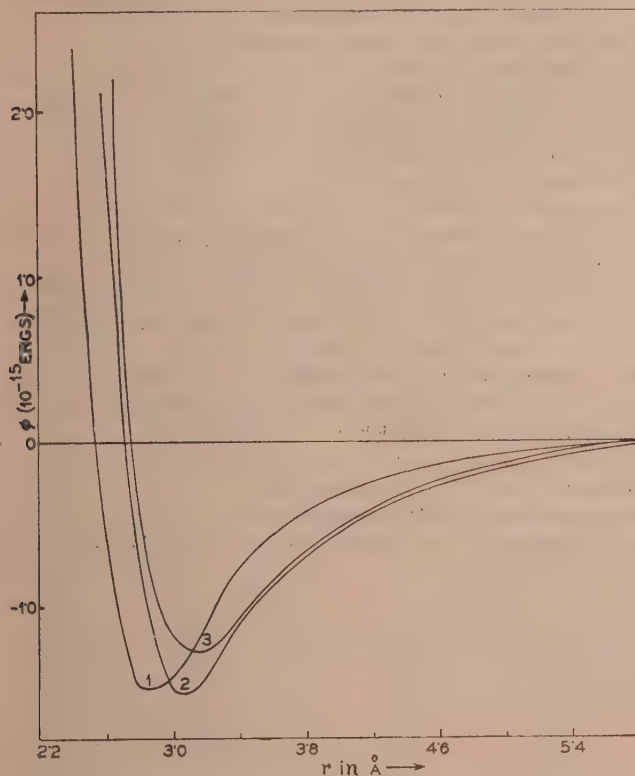
Fig. 1. Potential energy curves for He–He interaction (1) theoretical (Sakamoto-Ishinguro-Margenau-Page) (2) exp–6–8 (Buckingham-Corner) (3) exp.–6 (Mason and Rice).

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M. U. F. FACTOR AND SOLAR ACTIVITY

C. S. R. RAO AND J. C. BHARGAVA

RESEARCH DEPARTMENT, ALL INDIA RADIO, NEW DELHI

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ABSTRACT. This paper deals with the variation of $M(3000)F_2$ factor with sunspot activity. The ionospheric data for Delhi and Ahmedabad for the period 1950 to 1958 have been considered. The analysis indicates that a fairly good linear relationship exists between $M(3000)F_2$ and sunspot number for both the places. A preliminary study of the variation of Y_m and h_o with sunspot activity (Ahmedabad) has also been made.

I. INTRODUCTION

The variation of $M(3000)\,F_2$—the maximum usable frequency factor for distance of 3000 kms for transmission through the F_2 layer—with sunspot activity has been studied recently by a few workers. Eyfrig (1951), following the variation of $M(3000)\,F_2$ (related to the layer height), has observed that there exists a linear relationship. Rawer (1952) has stated that the variation of the factor $M(3000)\,F_2$ can be taken as an index of altitude of the layer and that it is correlated with sunspot number. Allen (1953), however, has concluded that the variation of virtual heights is not related in any significant manner to either the sunspot cycle or the diurnal variation of f_0F_2. Theissen (1955) has attempted the evaluation of $M(3000)\,F_2$ for every hour by use of charts and has also considered the possibility of a world-wide forecast of these factors. Eyfrig (1957) has followed up his previous study and has shown that an unambiguous relation of $M(3000)\,F_2$ with change of solar activity exists for certain regions of the earth. He has, however, stressed therein, that the data is insufficient and sometimes contradictory to justify a world-wide examination.

The object of this paper is to study the variation of $M(3000)F_2$ with sunspot activity as observed from the data at Delhi and Ahmedabad—two Indian stations where ionospheric sounding has been in progress for the last few years. For Delhi ($28°35'$N, $77°5'$E) data for years 1950 to 1958 and Ahmedabad ($23°0$ N. $72°40'$E) data for the years 1953 to 1958 have been analysed.

II. CALCULATION OF $M(3000)F_2$

Over the last few years, a detailed analysis of the structure of the F_2 layer has been made from the data from the different ionospheric stations in India. This analysis has shown that the assumption of a parabolic layer is a good approximation and for $Y_m/h_o \geqslant 0.2$ (where Y_m is the semi-thickness of the layer and h_o

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is the height of the lower edge of the F_2 layer), the variation of the factor $M(3000)F_2$ with hpF_2 (height of maximum ionization density and equal to the height at 0.834 times $f_0 F_2$) can be represented by a curve, which appears to be a hyperbola (Fig. 1). This curve is obtained from a mass plot of $M(3000)F_2$ factors, (after
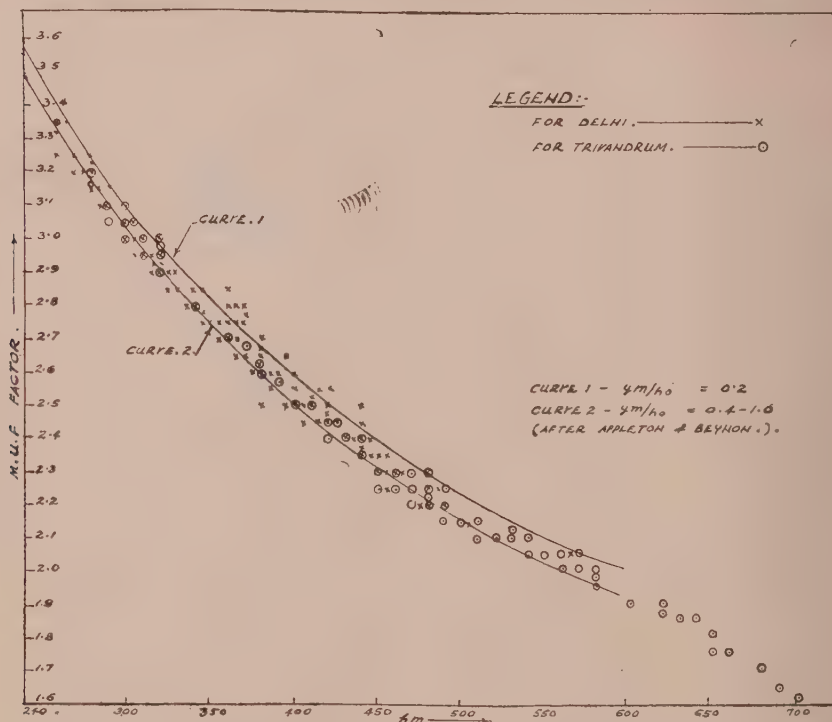

Fig. 1.   Variation of $M(3000)F_2$ with h_m for Delhi and Trivandrum.

Appleton and Beynon 1941, 1947) against height of maximum ionization for different values of $M(3000)F_2$. Bibl and others (1951) have also stated that hp F2 may be related to $M(3000)F_2$ and that the curve expressing the relationship appears to be a hyperbola. The curve corresponding to $Y_m/h_o = 0.4$ in Fig. 1 fits in best with the data for Indian stations.

All the $M(3000)F_2$ factors used in this paper have been obtained on the above basis.

The results of the analysis are discussed in the following sections.

III. DIURNAL AND SEASONAL VARIATIONS OF M(3000)F$_2$

Fig. 2 shows the diurnal variation of $M(3000)F_2$ for Delhi and Ahmedabad for sunspot minimum and maximum activity periods and for different seasons.

The predominance of the semidiurnal effect in the different seasons and in the different epochs of sunspot activity is quite evident. The effect of Solar tides in producing semi-diurnal variations of f_0F_2 and h_pF_2 has already been described earlier (Rao, 1956).
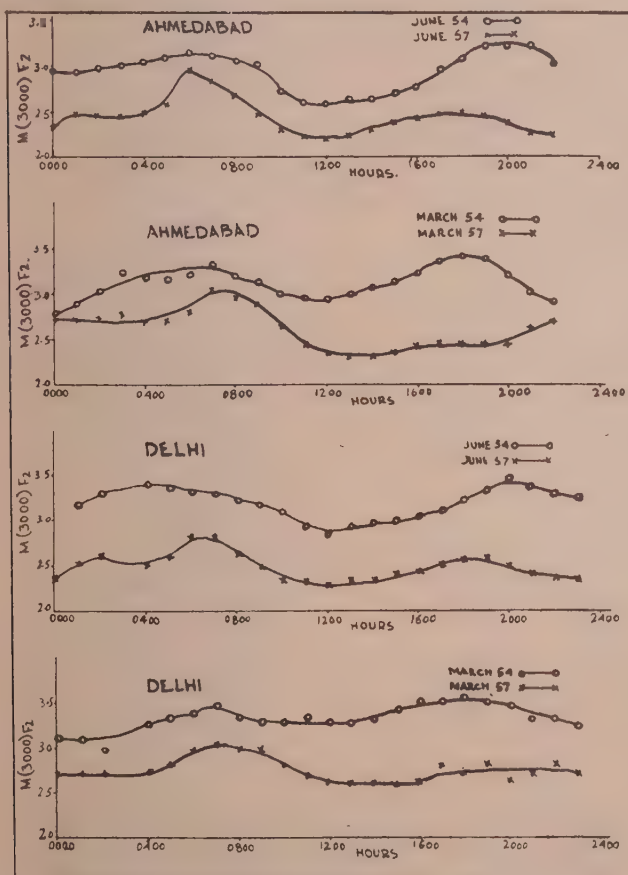
Fig. 2. Diurnal Variation of $M(3000)F_2$ for Delhi and Ahmedabad.

IV VARIATION OF 'M' WITH SUNSPOT ACTIVITY

Fig. 3 shows the variations of the 12 monthly running averages of $M(3000)$ F_2 as observed at Delhi and Ahmedabad with the 12 monthly running average of relative sunspot numbers for 1200 hrs. A study of these graphs indicates that a linear relationship between the $M(3000)F_2$ and sunspot number (R) would be a good approximation.

Assuming a relation of the type

$$M_R = M_0 - bR \qquad \qquad \cdots \quad (1)$$
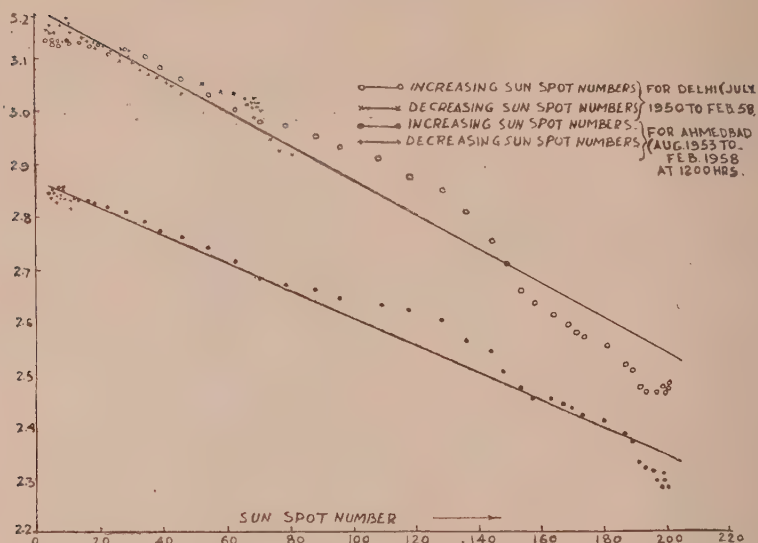
Fig. 3.　Variation of running average of $M(3000)F_2$ with running average Sunspot number.
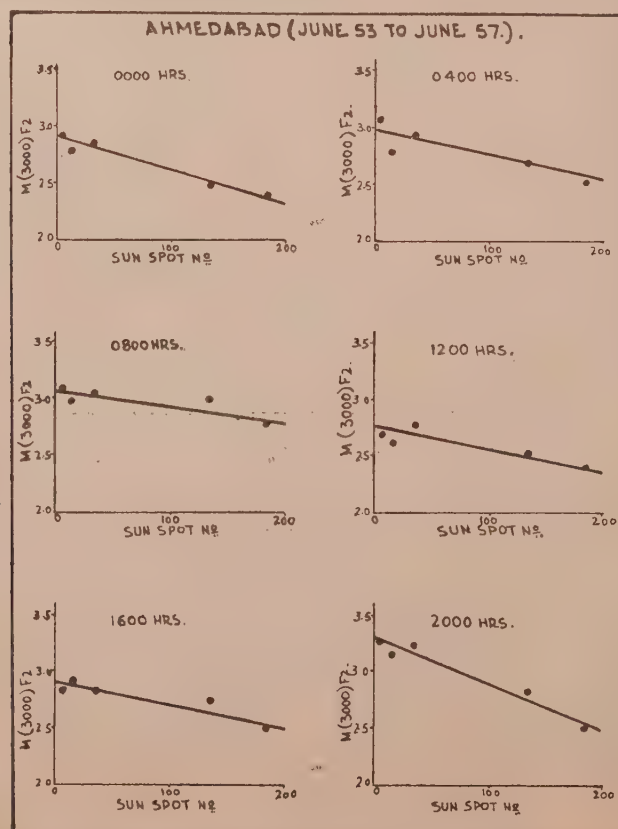
Fig. 4.　Variation of $M(3000)F_2$ with Sunspot number for different hours for Ahamedabad.

where M_R is the $M(3000)F_2$ factor for a sunspot number R, M_0 the same quantity for sunspot number zero and b the sunspot variation factor, the values of M_0

Fig. 5.   Variation of $M(3000)F_2$ with Sunspot number for different hours for Delhi.

and b and correlation co-efficient r between M_0 and R have been obtained and are shown in Table I.

TABLE I

Station	M_0	b	r
Delhi	3.22	0.00360	0.990
Ahmedabad	2.87	0.00266	0.971

The slow variation of M with sunspot number is apparent from the values given in the above table and the high values of r obtained justify the linear relationship assumed in Eq. 1.
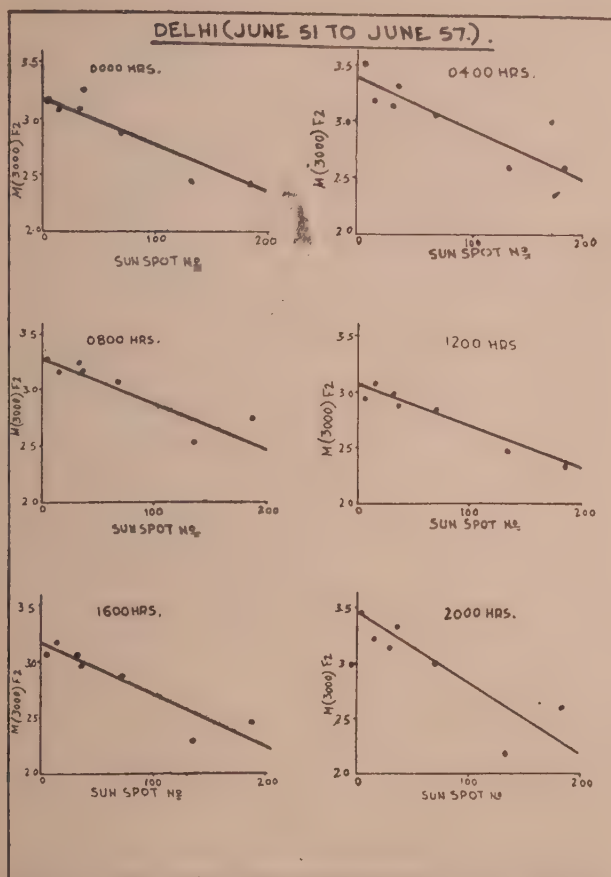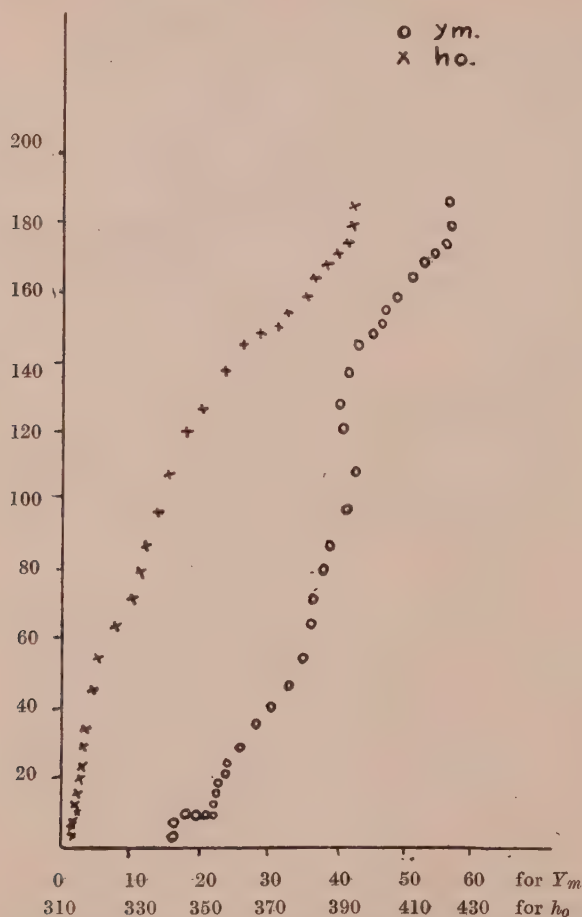

Fig. 6.   Variation of Y_m and h_o with Sunspot number for Ahmedabad.

The variation of $M(3000)\ F_2$ with sunspot number has also been studied for other hours.   A few typical graphs showing the variations for Delhi and Ahmedabad for the month of June for different hours of the day are shown in Figs. 4 and 5 respectively.   From these figures it is clear that the linear relationship assumed in Eq (1) can be extended to other hours of the day also.

V.   VARIATION   OF Y_m AND h_o

An initial study of the variation of Y_m and h_o of the F_2 layer at Ahmedabad with sunspot activity has been made.   The data have been taken from those published by the Physical Research Laboratory, Ahmedabad.   The variations

of Y_m and h_o (running average values of 12 months) with R are shown in Fig. 6, from which it may be seen that both Y_m and h_o increase in a linear manner with increase in R.

From these graphs it is seen that the same ratio of Y_m/h_o is maintained throughout the sunspot cycle.

ACKNOWLEDGMENT

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FOURTH ORDER MESON EQUATION AND NEUTRON-PROTON SCATTERING

S. P. MISRA

MATHEMATICS DEPARTMENT, RAVENSHAW COLLEGE, CUTTACK-3

ABSTRACT. We have obtained here the neutron-proton differential scattering cross-section with a fourth order meson equation proposed by Bhabha and Thirring, which was useful in explaining the anomalous magnetic moments of nucleons. We note that for moderate energies the results here disagree as violently with experiments as for conventional meson theory satisfying the Klein-Gordon equation.

INTRODUCTION

It was noted in a previous paper (Misra and Deo, 1956) that the treatment of the anomalous magnetic moments of nucleons with a fourth order meson equation proposed by Bhabha (1950) and Thirring (1950) gives comparatively satisfactory agreement with experimental results. While proposing this equation, Bhabha had shown that the second order potential derived from this does not have the r^{-3} singularity. Because of this advantage and its previous success, we shall calculate the neutron-proton scattering cross-section in the second order perturbation theory.

The interaction hamiltonian in this case is given as

$$H_i(x) = if \, \bar{\psi}(x) \gamma_5 \tau_k \psi(x) \phi_k(x). \qquad \dots \quad (1)$$

The interaction representation field-operators satisfy the equations

$$(\gamma_\mu \partial_\mu + \kappa_0)\psi(x) = 0,$$
$$\partial_\mu \bar{\psi}(x)\gamma_\mu - k_0 \bar{\psi}(x) = 0$$

and

$$(\square - \kappa^2)^2 \phi_k(x) = 0$$

The vacuum-expectation values of the P-products of the field operators are

$$<P(\phi_i(x)\phi_j(y))>_0 = \frac{\kappa^2}{2}\,\delta_{ij}D_F(x-y) \qquad \dots \quad (2)$$

where

$$D_F(x) = -\frac{2i}{(2\pi)^4}\int \frac{\exp{(ikx)}}{(k^2+\kappa^2)^2}\,d^4k \qquad \dots \quad (2a)$$

and

$$<P(\psi(x)\bar{\psi}(y))>_0 = \frac{1}{2} \ S_F(x-y) \qquad \dots \ (3)$$

where

$$S_F(x) = \frac{2i}{(2\pi)^4} \int \frac{i\gamma k + \kappa_0}{(k^2 + \kappa_0{}^2)} \ \exp \ (ikx)d^4k \qquad \dots \ (3a)$$

As usual, integrals (2a) and (3a) are to be understood in the sense of Feynman.

In the perturbation calculations, the S-matrix is given as

$$S = 1 + \overset{\infty}{\underset{n=1}{\Sigma}} \ S_n$$

where

$$S_n = (-i)^n/(n!) \int d^4x_1 \ \dots \ d^4x_n P(H_i(x_1) \ \dots \ H_i(x_n)).$$

SECOND ORDER S-MATRIX ELEMENT AND NEUTRON-PROTON SCATTERING

In the following, whenever we use the momentum space, p_1 and p_2 denote the four-momenta of the incoming proton and neutron and p_3 and p_4 denote the four-momenta of the outgoing proton and neutron respectively. Also, we use the expansion

$$\psi(x) = \frac{1}{(2\pi)^{3/2}} \int d^3p \sqrt{\frac{\kappa_0}{p_0}} \{a_p u(p)e^{ipx} + b_p{}^*v(p)e^{-ipx}\} \qquad \dots \ (4)$$

for the Dirac field operator, where a_p and $b_p{}^*$ represent respectively the annihilation and the creation operators of the particles and anti-particles.

The two Feynman diagrams that contribute to the second order matrix element are shown in Figs. 1(a) and 1(b).

(a)   1(b)
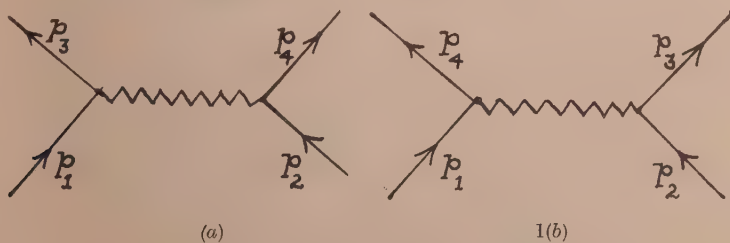
Fig. 1

These contribute to the second order matrix element, on using the expansion (4) and the results (2a) and (3), as,

$$<S_2> = -\frac{if^2\kappa^2}{(2\pi)^2} \ \delta(p_1 + p_2 - p_3 - p_4)\kappa_0{}^2(p_{10}p_{20}p_{30}p_{40})^{-\frac{1}{2}}$$

5

$$[\bar{u}(p_3)\gamma_5\tau_i u(p_1)\bar{u}(p_4)\gamma_5\tau_i u(p_2)((p_1 - p_3)^2 + \kappa^2)^{-2}$$

$$-\bar{u}(p_3)\gamma_5\tau_i u(p_2)\bar{u}(p_4)\gamma_5\tau_i u(p_1)((p_2 - p_3)^2 + \kappa^2)^{-2}]. \qquad \dots \text{ (5)}$$

On carrying out summation over isotopic spin indices for the proton and neutron states, we obtain,

$$<S_2> = \frac{if^2 k^2}{(2\pi)^2}\,\delta(p_1 + p_2 - p_3 - p_4)\kappa_0^2(p_{10}p_{20}p_{30}p_{40})^{-\frac{1}{2}}$$

$$[\bar{u}(p_3)\gamma_5 u(p_1)\bar{u}(p_4)\gamma_5 u(p_2)((p_1 - p_3)^2 + \kappa^2)^{-2}$$

$$+ 2\bar{u}(p_3)\gamma_5 u(p_2)\bar{u}(p_4)\gamma_5 u(p_1)((p_2 - p_3)^2 + \kappa^2)^{-2}]. \qquad \dots \text{ (6)}$$

Now, we know that if the S-matrix element between the initial state $|i>$ and the final state $|f>$ is given as

$$<S> = \delta(P_i - P_f) <f|M|i> \qquad \dots \text{ (7)}$$

where P_i and P_f are the total initial and final four-momenta respectively, then the total scattering cross-section is given as

$$\sigma = (2\pi)^2 \frac{p_{10}p_{20}}{((p_1 p_2)^2 - \kappa_0^4)^{\frac{1}{2}}}\, \bar{S}_i\, S_f\, \{\delta(P_i - P_f)\,|<f|M|i>|^2\}, \qquad \dots \text{ (8)}$$

(Jauch and Rohrlich (1955)), where S_i stands for averaging over the initial states and S_f stands for the summation over all the final states. When we utilise the centre of mass coordinates, the equation (8) gives us the differential scattering cross-section as

$$\frac{d\sigma}{d\Omega} = \pi^2 E^2 \bar{S}_i\, \Sigma\, |<f|M|i>|^2, \qquad \dots \text{ (9)}$$

where Σ stands for summation over final spin states.

For our problem, equations (6) and (7) give us

$$<f|M_2|i> = \frac{if^2 k^2}{(2\pi)^2}\,\frac{k_0^2}{E^2}\,[\,\bar{u}(p_3)\gamma_5 u(p_1)\bar{u}(p_4)\gamma_5\, u(p_2)\,((p_1 - p_3)^2 + \kappa^2)^{-2}$$

$$+ 2\bar{u}(p_3)\gamma_5 u(p_2)\bar{u}(p_4)\gamma_5 u(p_1)((p_2 - p_3)^2 + \kappa^2)^{-2}]. \qquad \dots \text{ (10)}$$

The spin summation in equation (9) can be performed by using the projection operator $\wedge_+(p)$ to the positive energy states given as

$$\wedge_+(p) = \frac{-i\gamma p + \kappa_0}{2E}\,\beta$$

Then we have, for equation (10),

$$S_i \ \Sigma \, | < f \, | \, M \, | i > |^2$$

$$= \frac{1}{2} \frac{f^4 \kappa^4}{(2\pi)^4} \frac{\kappa_0^4}{E^4} \ [((p_1 - p_3)^2 + \kappa^2)^{-4} \ Sp(\beta\gamma_5 \wedge_+(p_1)\gamma_5\beta \wedge_+(p_3)] \times$$

$$\times Sp[\beta\gamma_5 \wedge_+(p_2)\gamma_5\beta \wedge_+(p_4)] + 4((p_2 - p_3)^2 + \kappa^2)^{-4} \ Sp[\beta\gamma_5 \wedge_+(p_2)\gamma_5\beta \wedge_+(p_3)] \times$$

$$\times Sp[\beta\gamma_5 \wedge_+(p_1)\gamma_5\beta \wedge_-(p_4)] + 2((p_1 - p_3)^2 + \kappa^2)^{-2} \ ((p_2 - p_3)^2 + \kappa^2)^{-2} \times$$

$$\times \{Sp[\beta\gamma_5 \wedge_+(p_1)\gamma_5\beta \wedge_+ p_4 \beta\gamma_5 \wedge_+(p_2) \quad \gamma_5\beta \wedge_+(p_3) + \text{hermitian} \quad \text{conjugate} \quad \text{expn.}\}]$$

$$= \frac{1}{2} \frac{f^4 \kappa^4}{(2\pi)^4} \frac{\partial^{\kappa^4}}{E^8} \ [((p_1 - p_3)^2 + \kappa^2)^{-4} \ ((p_1 p_3) + \kappa_0^2) \ ((p_2 p_4) + \kappa_0^2)$$

$$+ 4((p_2 - p_3)^2 + \kappa^2)^{-4} \ ((p_2 p_3) + \kappa_0^2) \ ((p_1 p_4) + \kappa_0^2)$$

$$+ ((p_1 - p_3)^2 + \kappa^2)^{-2} \ ((p_2 - p_3)^2 + \kappa^2)^{-2} \{(p_1 p_4)(p_2 p_3) - (p_1 p_2)(p_3 p_4) + (p_1 p_3)(p_2 p_4)$$

$$+ \kappa_0^2((p_1 p_4) - (p_1 p_2) + (p_1 p_3) + (p_2 p_4) + (p_2 p_3) - (p_3 p_4)) + \kappa_0^4\}]. \qquad \dots \ (11)$$

We now remember that for the centre of mass system,

$$p_1 = (\overrightarrow{P}, E), \qquad p_2 = (-\overrightarrow{P}, E)$$

$$p_3 = (\overrightarrow{P'}, E), \qquad p_4 = (-\overrightarrow{P'}, E), \qquad \qquad \dots \ (12)$$

and substitute

$$\overrightarrow{P'} . \overrightarrow{P} = P^2 \cos\theta \qquad \qquad \dots \ (13)$$

This gives us, on simplification of the above equation (11) and by equation (9),

$$\frac{d\sigma}{d\Omega}$$

$$= \left(\frac{f^2}{4\pi}\right)^2 \frac{2\kappa^4\kappa_0^4}{E^6 P^4} \left[\frac{\sin^4 \frac{\theta}{2}}{\left(4 \sin^2 \frac{\theta}{2} + \lambda\right)^4} + \frac{4 \cos^4 \frac{\theta}{2}}{\left(4 \cos^2 \frac{\theta}{2} + \lambda\right)^4} \right.$$

$$\left. - \frac{2 \sin^2 \frac{\theta}{2} \ \cos^2 \frac{\theta}{2}}{\left(4 \sin^2 \frac{\theta}{2} + \lambda\right)^2 \left(4 \cos^2 \frac{\theta}{2} + \lambda\right)^2} \right] \qquad \dots \ (14)$$

Thus, for moderate energies, the differential scattering cross-section is given as

$$\frac{d\sigma}{d\Omega} = \left(\frac{f^2}{4\pi}\right)^2 \frac{2}{k_0^2} \ g(\theta)$$

where

$$g(\theta) = \lambda^2 \left[\frac{\sin^4 \frac{\theta}{2}}{\left(4 \sin^2 \frac{\theta}{2} + \lambda \right)^4} + \frac{4 \cos^4 \frac{\theta}{2}}{\left(4 \cos^2 \frac{\theta}{2} + \lambda \right)^4} \right.$$

$$\left. - \frac{2 \sin^2 \frac{\theta}{2} \cos^2 \frac{\theta}{2}}{\left(4 \sin^2 \frac{\theta}{2} + \lambda \right)^2 \left(4 \cos^2 \frac{\theta}{2} + \lambda \right)^2} \right],$$

with $\lambda = (\kappa^2/P^2)$. The laboratory energy is given here approximately as $(42/\lambda)$ Mev.

Except for a multiplicative constant, the function $g(\theta)$ above gives us the cross-section, and we have plotted this against θ for $\lambda = 1$ and $\lambda = (1/2)$, i.e.. for laboratory energies 42 MeV and 84 MeV respectively (Fig. 2). It is noted that the nature of the curve obtained does not agree with the well-known experimental form, which should be rather symmetrical about 90° with a minimum at slightly less than that value. On the other hand, up to angle 140° the angular distribution is rather suggestive of the behaviour at higher energies. This leads to the suspicion that a smaller value of $(1/\lambda)$ may give the experimental type of curve for lower energies. But in such cases the peak of the curve comes too near 90° for these to have any significance.
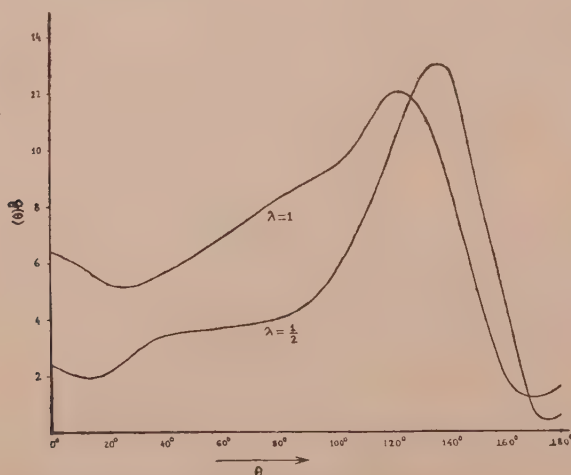

Fig. 2.   The function of $g(\theta)$ is plotted against the centre of mass angle θ at laboratory energies 42 Mev ($\lambda = 1$) and 84 MeV ($\lambda = 0\cdot5$).

A rough comparison of the scattering data with experiments gives. (on using 1 Mev^{-1} = 1.973 10^{-11} cms.), that,

$$\frac{f^2}{4\pi} \simeq 16$$

Thus we note that this meson theory disagrees with experimental results for neutron-proton scattering as violently as the conventional one. Hence the fairly good agreement (Misra and Deo,) (1955)* that was obtained for the treatment of the anomalous magnetic moments of nucleons may be regarded as accidental.

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*There has been unfortunately a mistake in this paper in taking the meson field Lagrangian. This, as taken in the paper, should be multiplied by 2 so that it gives rise to the propagator quoted. This correction also gives better agreement with experiments with $|\Delta^\mu{}_N / \Delta^\mu{}_P| = 1.6$ instead of 2.5.

Letters to the Editor

2

SEARCH FOR EO TRANSITION IN ZINC-68

M. K. RAMASWAMY

DEPARTMENT OF PHYSICS, THE OHIO STATE UNIVERSITY COLUMBUS-10, OHIO (USA)

(Received, January 29, 1960)

The decay of Ga-68 has been recently reinvestigated by Horen (1959) who found evidence for levels in Zinc-68 at 1.07, 1.88 and 2.3 Mev. The levels were assigned the following spins and parities : 1.07(2+), 1.88 (1+ or 2+) and 2.3(2+). No cross-over gamma transition was observed from the 2.3 Mev level to the 0+ ground state. This fact together with the result of angular correlation studies on the 1.07-1.24 Mev gamma cascade led Horen to suggest 2+ to the 2.3 Mev level. Since the errors on the angular correlation data were rather large, this spin assignment was considered to be far from ambiguous. It seemed attractive to assign 0+ to this state and still be consistent with the observed data. In fact, the near harmonic model of Scharff-Goldhaber and Weneser (1955) predicts the existence of 0+ levels at about twice the energy of the first excited state.

0—0. No gamma transitions are strictly forbidden because of the transverse nature of the photon. Hence the de-excitation of the state in question can proceed either through internal conversion or through internal pair formation. Since the internal positron spectrum from the 2.3 Mev level (end-point 1.28 Mev) would be superposed on a rather high background of positrons from the beta decay of Ga-68 (end-point 1.88 Mev), it was decided to look for internal conversion electrons of 2.3 Mev. In order to observe the small peak due to conversion it was necessary to reduce the background to a minimum. This was accomplished by the use of an anti-coincidence arrangement.

The Ga-68 source was sandwiched between two plastic scintillators each 1-1/2 cm high and 3 cm in diameter ensuring 4π geometry, and mounted on the face of a Du Mont 6292 phototube. A second plastic scintillator surrounding the first one was mounted on another phototube and served as the anti-coincidence

counter. The sensitive volume of the counter was surrounded by 4″ of lead. Pulses in counter 1 not accompanied by pulses in counter 2 operated a gate which admitted to the 20-channel analyzer any pulse in the beta-counter occurring in an interval of 3.5 microseconds. The end-point of the positron spectrum was determined to be 1.89 ± 0.05 Mev, in agreement with the value reported by Daniel (1957). Phosphorus-32 with end-point energy of 1.72 Mev(Lidofsky, 1957) served as the calibration spectrum. The search for internal conversion electrons lasted 105 hours, during which time the apparatus was periodically checked. From the total number of counts observed in the region where the conversion electrons were expected and the total number of counts in the beta-spectrum, an upper limit of $(5 \pm 25) \times 10^{-9}$ conversion electrons per Ga-68 decay could be set.

One can calculate the expected yield of 2.3 Mev internal conversion electrons as follows (assuming 0^+ level). The single-particle model gives the transition probability for the 1.24 Mev gamma-ray as $W_{1 \cdot 28} = 10^{11}$ sec^{-1}. The K-conversion probability for the Eo transition is expressed by Church and Weneser (1956) as $W_k/\rho^2 = 7 \times 10^8$ sec^{-1} where ρ^2 is a dimensionless parameter which measures the strength of the EO matrix element. We take $\rho \sim 0.11$ in accordance with experiment (Alburger, 1958). The yield can then be written

$$N_k = \left[(W_k/\rho^2)\rho^2 / \left(\frac{W_k}{\rho^2} \rho^2 + W_{1 \cdot 24} \right) \right] fN$$

where N is the total number of disintegrations and f is the fraction of decays populating the 2.3 Mev level. From the data of Horen and the measured beta-spectrum Nf is about 2×10^5. The computed yield is 84×10^{-9} conversion electrons per decay, to be compared with the measured value $(5 \pm 25) \times 10^{-9}$.

If these estimates are indeed correct, then one can conclude that 0^+ is an unlikely assignment for the 2.3 Mev level. If, on the other hand, it turns out that the 1.24 Mev transition is 10 times faster than the single-particle estimate, by analogy with the situation in Ga-70, then our conclusions are somewhat weakened. Recent angular correlation measurements (Ramaswamy and Jastra, 1960) have indeed shown that the 2.3 Mev level is 2^+.

The author is thankful to Professor L. Madansky of the Johns Hopkins University where this work was performed.

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BOOK REVIEW

THEORIES OF FIGURES OF CELESTIAL BODIES—by W. S. Jardetzky.
   Pages 186; 6×9 in. Inter-science Publishers—New York—London, 1958.
   Price $ 6.50.

One of the most important problems of any system of bodies is its evolution, that is, the sequence of changes it undergoes in time under the influence of its surroundings and internal variations. In the case of celestial bodies the problem has attracted the attention of a number of scientists. There are in general two fronts on which the problem is attacked. One is the physical state in which the study of internal constitution describes the run of the physical variables at different points inside the body. The other is the hydro-dynamical and kinematical state which describes the geometrical forms that a fluid body should acquire under the influence of the internal and external forces. In both these cases the fast varying catastrophic changes are not included. It is assumed that the results conform to the situation which is established in due course of time when the final equilibrium is attained. For celestial objects it has become quite clear especially during recent years that the two fronts have to come very close and work together. Convection plays an important role in the study of the internal constitution. It is highly probable that the rotation is only a part of the general circulation of matter in a celestial body due to rotation as well as convection. It is believed in general that the probability of existence of a celestial body without currents of matter in its interior is practically zero. The general problem at present is still far from any complete solution.

On the interior side the active front was opened only during the present century. But the work on the dynamical front has been going on since the realization of the importance of Newton's Law of Gravitation. During these centuries quite a rigorous mathematical theory has been developed for the figures of the bodies in equilibrium by a number of great scientists like Maclaurin, Jacobi, Clairaut, Poincare', Liapounov, Lichtenstein, Wavre, Mineo, Jardetzky and others. All these studies are related to the equilibrium configurations under different forms of rotation for the bodies consisting of isolated, homogeneous or heterogeneous fluid masses or composite solid and fluid masses. Binaries and other complicated systems are also covered. Finally, this branch also deals with the possible sequence of equilibrium figures as a result of externally undisturbed evolution. In the last case possible atomic energy production inside the body cannot be taken into account as these studies are confined to hydrodynamical situations only under conservative forces, the forces depending upon velocities and the gyroscoptical forces.

A general study of figures of equilibrium of fluid masses under the above mentioned forces is a problem of utmost importance specially to the celestial

mechanics. To have an idea of the importance of this branch we may note that when we look around we find that almost every thing in the universe rotates. The meteors, the asteroids, the planets, their satellites, the sun, the stars, the clusters, the nebulae and even the galaxies-all rorate in such a way that they have acquired over a long time a definite equilibrium distribution of velocities and a geometrical shape. Irregular forms of certain planetoids or nebulae are rather exceptions and the origin of these must be sought in some type of forces other than simple gravitation. The present position of the study of these dynamical conditions relates to rotation, pulsation, oscillations of some other type, progressing variations and rotation and revolution of two or more bodies. It is quite obvious that the problem of figures of equilibrium, though important, is quite difficult one. Some of them need very advanced mathematical tools for analysis. To-day the two important fields in this direction are : the small deformations of a gaseous mass and its stratification which have application in stars and the variation in viscosity, plasticity and the period of solidification which are important for the theories of the cold bodies like the planets. Observational confirmation of the theory can be available only for the limited cases of members of the solar system. The zonal rotation is important because it is actually observed in the case of planets like Jupiter and Saturn and also for the Sun. Law of variation of this zonal rotation from poles to the equator still remains an empirical one.

In view of what has been said above the book "Theories of Figures of Celestial Bodies" by W. S. Jardetzky, who has himself contributed a lot to this field, makes a very good addition to the existing literature, all the more so because majority of the original papers are in languages other than English. The book is well written. It is equally important to and can be appreciated by both beginners and advanced students in the field. In this book the phrase "equilibrium figures, of a body", has been used in a more general sense which includes the stratification. In the first part, in a few chapters, the author has developed the mathematical theory according to historical sequence. In the second part applications of the theory are made for explaining observed situations for the earth as well as other planets. The probable explanation of Saturn's rings is also suggested. The stratification studies which are applicable to stars and for which no observational check is possible has been dealt summarily. A brief account of the system of binaries is also given.

The order of the material is excellent. The book contains a bibliography of almost exhaustive references for further details on the subject. The study of this book can form a very good background for any student who wishes to work in this particular field. The printing and the quality of the paper make the book easy reading.

R. S. K.

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